Physicsaholics

DPP – 2

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Written Solution on Website:- https://physicsaholics.com/note/notesDetalis/49

Q 1. Calculate the center of mass of a non-uniform rod whose linear mass density (λ) varies as $\lambda = \frac{\lambda_0}{L}x^2$, where λ_0 is a constant, L is the length of the rod and x distance is measured from one end of the rod

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$

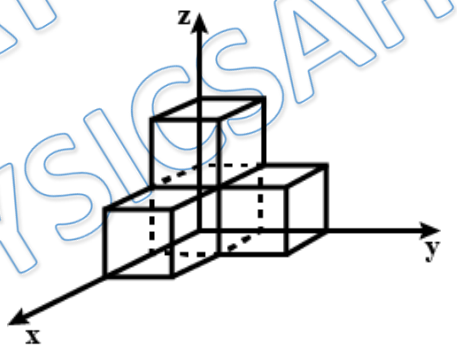
(c) $\frac{3L}{4}$ (d) $\frac{L}{3}$

Q 2. A non-uniform thin rod of length l lies along the axis with one end at the origin. It has a liner mass density $\lambda = \lambda_o\left(1 + \frac{x}{l}\right)$. Find the center of mass of the rod

(a) $\frac{2l}{9}$ (b) $\frac{5l}{9}$

(c) $\frac{2l}{5}$ (d) $\frac{3l}{7}$

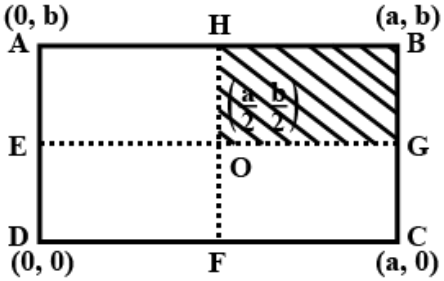
Q 3. Find the center of mass (x, y, z) of the following structure of four identical cubes if the length of each side of a cube is 1 unit



(a) $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

(c) $\left(\frac{3}{4},\frac{3}{4},\frac{3}{4}\right)$ (d) $\left(\frac{1}{2},\frac{3}{4},\frac{1}{2}\right)$

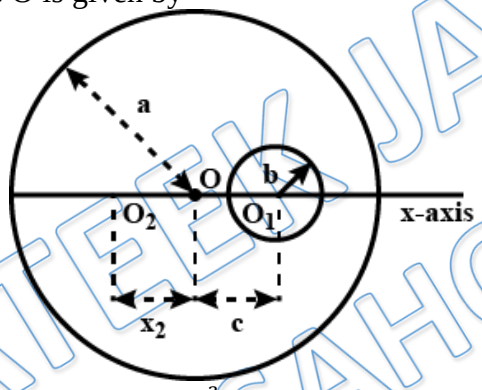
Q 4. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the center of mass of the remaining portion will be :-



(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

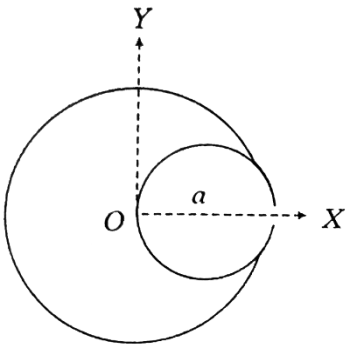
Q 5. A uniform circular disc of radius a is taken. A circular portion of radius b has been removed from it as shown in the figure. If the center of hole is at a distance c from the center of the disc, the distance x_2 of the center of mass of the remaining part from the initial center of mass O is given by



(a) $\frac{\pi b^2}{a^2-c^2}$ (b) $\frac{cb^2}{a^2-b^2}$

(c) $\frac{\pi c^2}{a^2-b^2}$ (d) $\frac{ca^2}{c^2-b^2}$

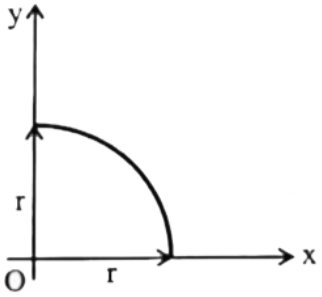
Q 6. Find the position of center of mass of the uniform lamina shown in figure, if small disc of radius $\frac{a}{2}$ is cut from disc of radius a. (Consider point 'O' as origin)



(a) (0, 0) (b) $\left(0, -\frac{a}{6}\right)$

(c) $\left(-\frac{a}{6}, -\frac{a}{6}\right)$ (d) $\left(-\frac{a}{6}, 0\right)$

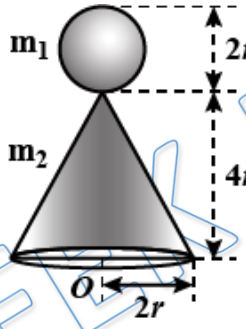
Q 7. The coordinates of the center of mass of the following uniform quarter circular arc are



(a) $\left(\frac{r}{2},\frac{r}{2}\right)$ (b) $\left(\frac{2r}{3},\frac{2r}{3}\right)$

(c) $\left(\frac{2r}{\pi},\frac{2r}{\pi}\right)$ (d) none of these

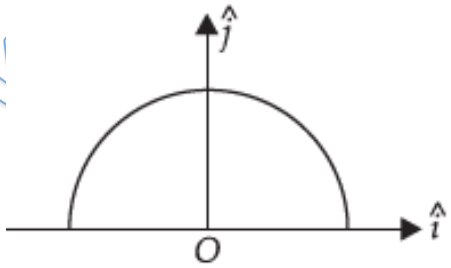
Q 8. A solid cone and a sphere is shown in the figure. The density of material of cone is 1/12 times that of the sphere. The distance of COM on the line of symmetry from O is:



(a) 4r (b) 3r

(c) 2r (d) 5r

Q 9. Find the center of mass of uniform semi-circular ring of radius R



(a) $\left(0,\frac{r}{2}\right)$ (b) $\left(0,\frac{r}{\pi}\right)$

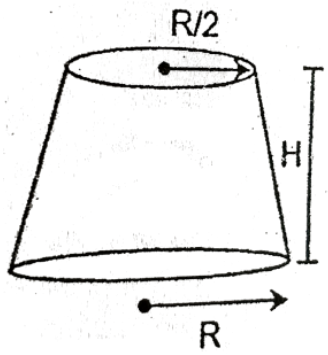
(c) $\left(0,\frac{2r}{\pi}\right)$ (d) $\left(0,\frac{2r}{3}\right)$

Q 10. A thin uniform wire is bent to form the two equal sides AB and AC of triangle ABC, where AB=AC=5 cm. The third side BC, of length 6cm, is made from uniform wire of same cross-section and twice the density of the first. The distance of the center of mass from A is

(a) $\frac{23}{11}$ cm (b) $\frac{34}{9}$ cm

(c) $\frac{12}{11}$ cm (d) $\frac{34}{11}$ cm

Q 11. A frustum of a uniform solid cone has base radius R and height H as shown. Radius of top surface is R/2. If height of center of mass of frustum is $\frac{11H}{4n}$ from base then n will be



(a) 2 (b) 3
(c) 7 (d) 9

Q 12. Two spheres of mass M and 7M are connected by a rod whose mass is negligible, and the distance between the centers of each sphere is d. How far from the center of the 7M sphere is the Center of Mass for this object?

(a) $\frac{d}{8}$ (b) $\frac{d}{7}$
(c) $\frac{d}{2}$ (d) $\frac{2d}{7}$

Answer Key

Q.1 c	**Q.2 b**	**Q.3 c**	**Q.4 d**	**Q.5 b**
Q.6 d	**Q.7 c**	**Q.8 a**	**Q.9 c**	**Q.10 d**
Q.11 c	**Q.12 a**			

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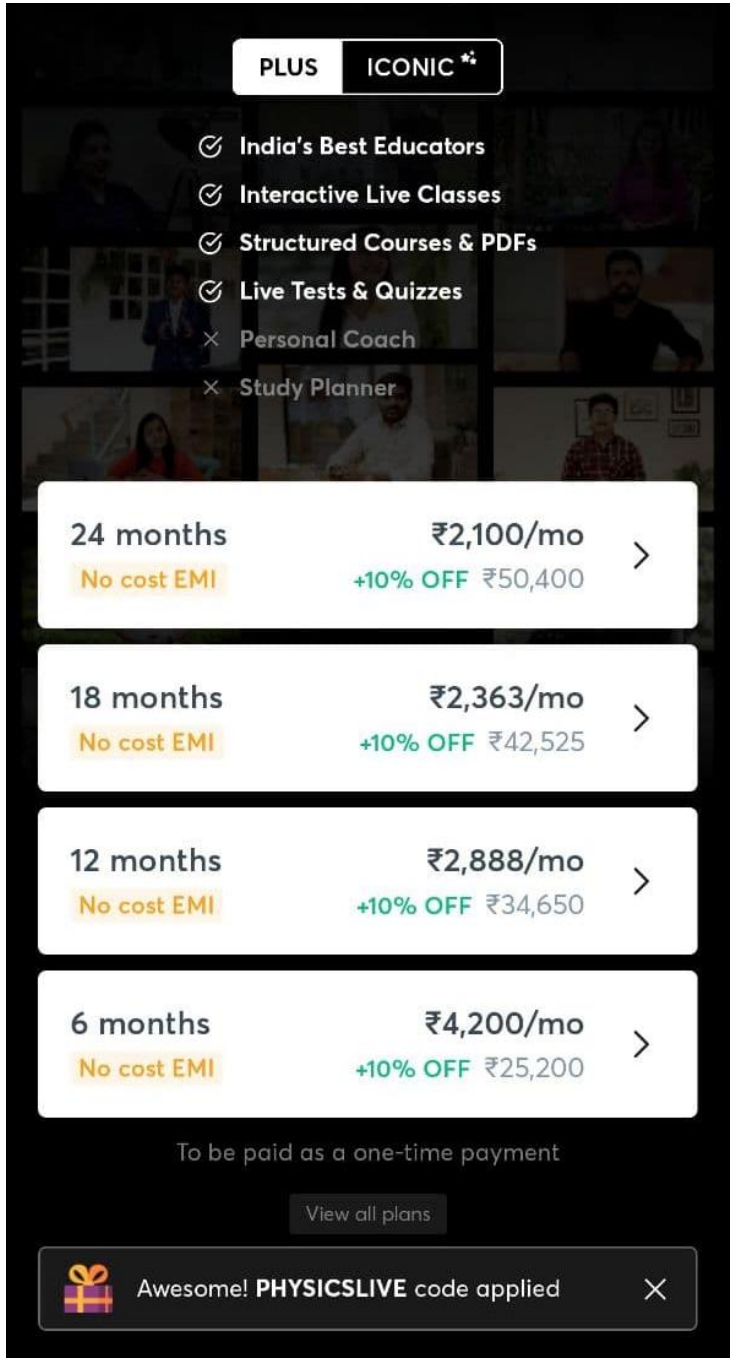


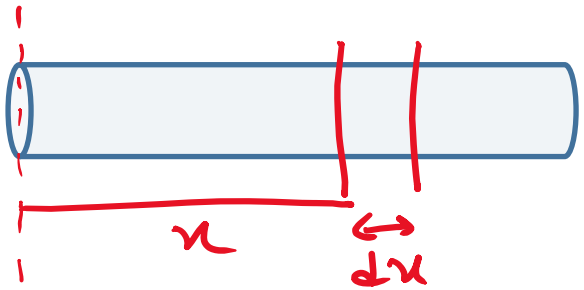
Written Solution

DPP-2 COM: COM of Continuous Mass System

By Physicsaholics Team

Solution: 1

For the element of very small length 'dx' $dm = \lambda\, dx$



$$dm = \left(\frac{d_0}{L}x^2\right)(dx)$$

So, $$x_{cm} = \frac{\int dm\, x}{\int dm} = \frac{\int_0^L x\left(\frac{d_0}{L}x^2 dx\right)}{\int_0^L \left(\frac{d_0}{L}x^2 dx\right)} = \frac{\int_0^L x^3 dx}{\int_0^L x^2 dx}$$

$$x_{cm} = \frac{\left(x^4/4\right)_0^L}{\left(x^3/3\right)_0^L} = \frac{\left[\frac{L^4}{4} - 0\right]}{\left[\frac{L^3}{3} - 0\right]} = \frac{3}{4}L$$

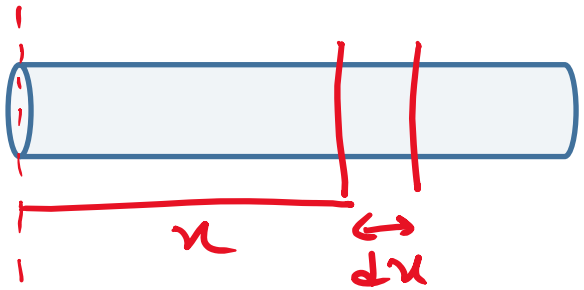
$$\Rightarrow \boxed{x_{cm} = \frac{3}{4}L}$$ Ans.

Ans. c

Solution: 2

For the element of very small length 'dx' $dm = \lambda = \lambda(dx)$

$$dm = \lambda_0\left(1+\frac{x}{l}\right)dx$$



So; $x_{cm} = \dfrac{\int dm\,x}{\int dm} = \dfrac{\int_0^l \lambda_0\left(1+\frac{x}{l}\right)dx \times x}{\int_0^l \lambda_0\left(1+\frac{x}{l}\right)dx} = \dfrac{\int_0^l \left(x+\frac{x^2}{l}\right)dx}{\int_0^l \left(1+\frac{x}{l}\right)dx}$

$$x_{cm} = \frac{\left[\frac{x^2}{2}+\frac{x^3}{3l}\right]_0^l}{\left[x+\frac{x^2}{2l}\right]_0^l} = \frac{\left[\frac{l^2}{2}+\frac{l^2}{3}\right]}{\left[l+\frac{l}{2}\right]} = \frac{\frac{5l^2}{6}}{\frac{3l}{2}} = \frac{5}{9}l$$

$$\boxed{x_{cm} = \frac{5}{9}l}\ \text{Ans.}$$

Ans. b

Solution: 3

COM of cube-1 : $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

COM of cube-2 : $\left(\frac{1}{2}, \frac{3}{2}, \frac{1}{2}\right)$

COM of cube-3 : $\left(\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

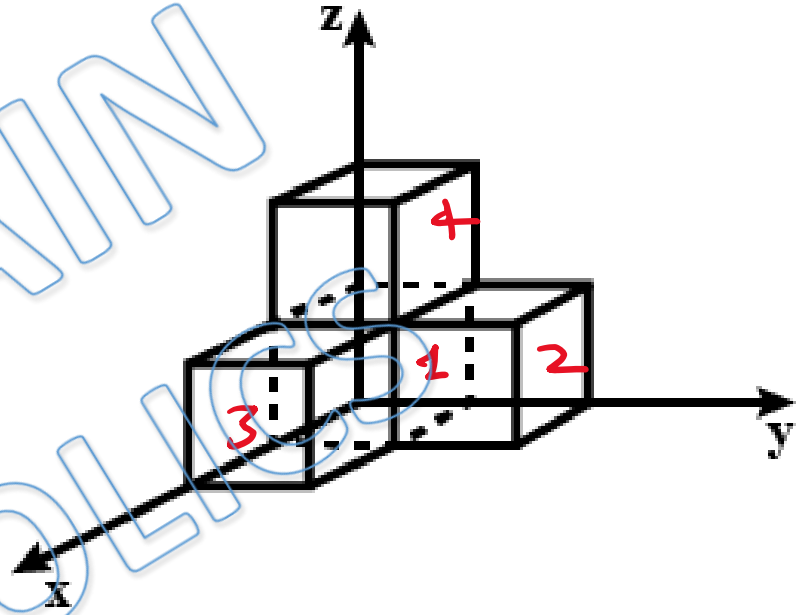
COM of cube-4 : $\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

$$\therefore x_{cm} = \frac{m\left(\frac{1}{2}\right) + m\left(\frac{1}{2}\right) + m\left(\frac{3}{2}\right) + m\left(\frac{1}{2}\right)}{4m}$$

$$x_{cm} = \frac{3m}{4m} = \frac{3}{4} \Rightarrow \boxed{x_{cm} = \frac{3}{4}}$$

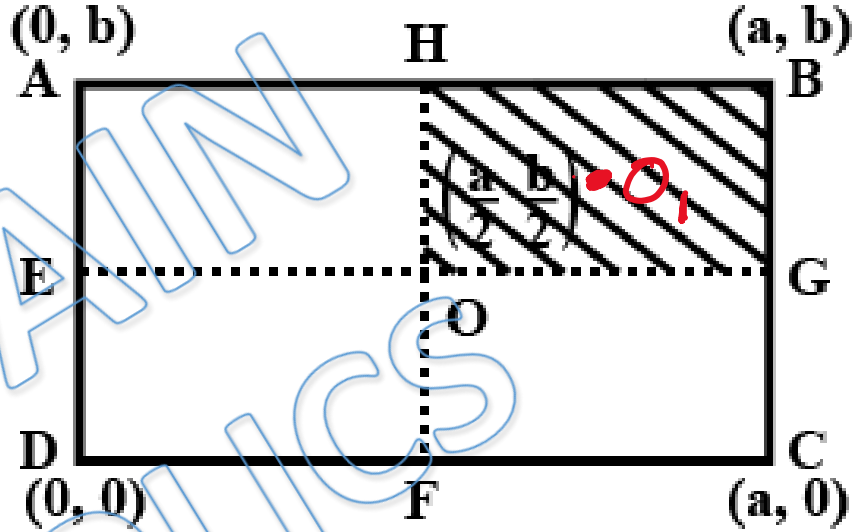
Similarly; $\boxed{y_{cm} = \frac{3}{4}}$ and $\boxed{z_{cm} = \frac{3}{4}}$

So; $\boxed{r_{cm} : \left(\frac{3}{4}, \frac{3}{4}, \frac{3}{4}\right)}$ Ans.



Ans. c

Solution: 4



Point O_1 = COM of Part OGBH

so; $O_1 : \left(\frac{3a}{4}, \frac{3b}{4}\right)$

Mass of Part OGBH; $m_1 = \frac{M}{4}$

so; Mass of remaining part $= m_2 = \frac{3M}{4}$

let Cordinate of com of remaining mass $= (x, y)$

For COM of Both parts $(x_{cm}, y_{cm}) : \left(\frac{a}{2}, \frac{b}{2}\right)$

$$\therefore \frac{a}{2} = \frac{\frac{M}{4}\left(\frac{3a}{4}\right) + \frac{3M}{4}(x)}{M} \Rightarrow \frac{a}{2} = \frac{3a}{16} + \frac{3x}{4} \Rightarrow \frac{3x}{4} = \frac{5}{16}a$$

$$\boxed{x = \frac{5}{12}a}$$; similarly : $$\boxed{y = \frac{5}{12}b}$$

$\therefore$ COM of remaining part : $\left(\frac{5}{12}a, \frac{5}{12}b\right)$ Ans

Ans. d

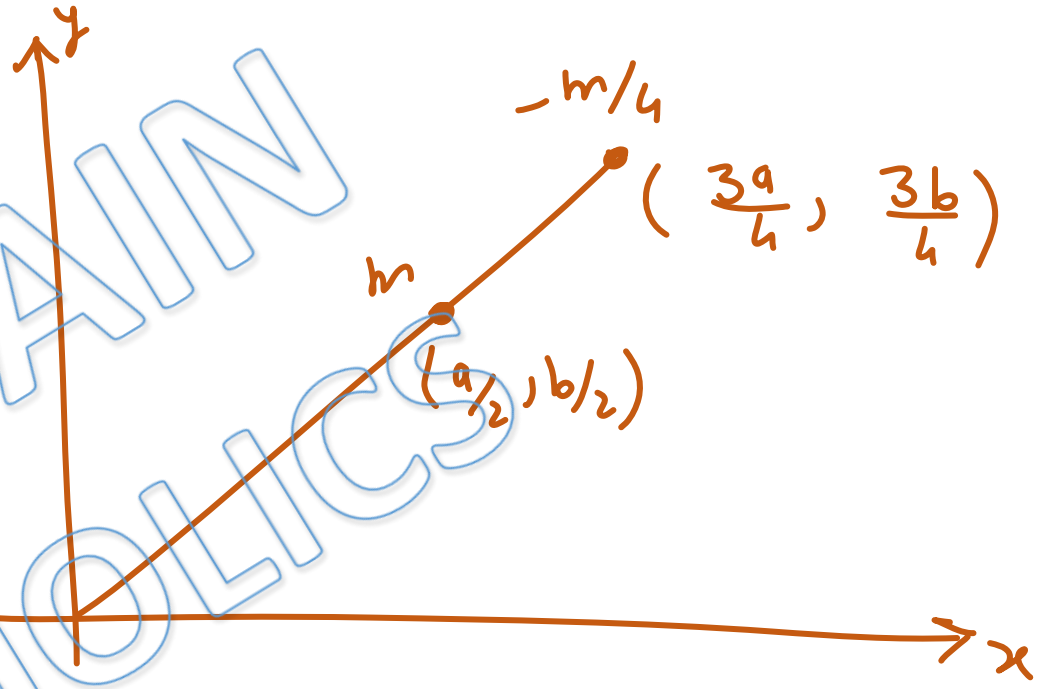
OR

Using -ve mass method

$$\vec{r}_{CM} = \frac{m\left(\frac{a}{2}\hat{i} + \frac{b}{2}\hat{j}\right) - \frac{m}{4}\left(\frac{3a}{4}\hat{i} + \frac{3b}{4}\hat{j}\right)}{m - m/4}$$

$$= \frac{\frac{5ma}{16}\hat{i} + \frac{5mb}{16}\hat{j}}{3m/4}$$

$$= \frac{5a}{12}\hat{i} + \frac{5b}{12}\hat{j}$$



Ans. d

Solution: 5

Let; mass of unit area of disc = k

$\therefore$ mass of hole; $m_1 = k\pi b^2$

and; mass of disc; $M = k\pi a^2$

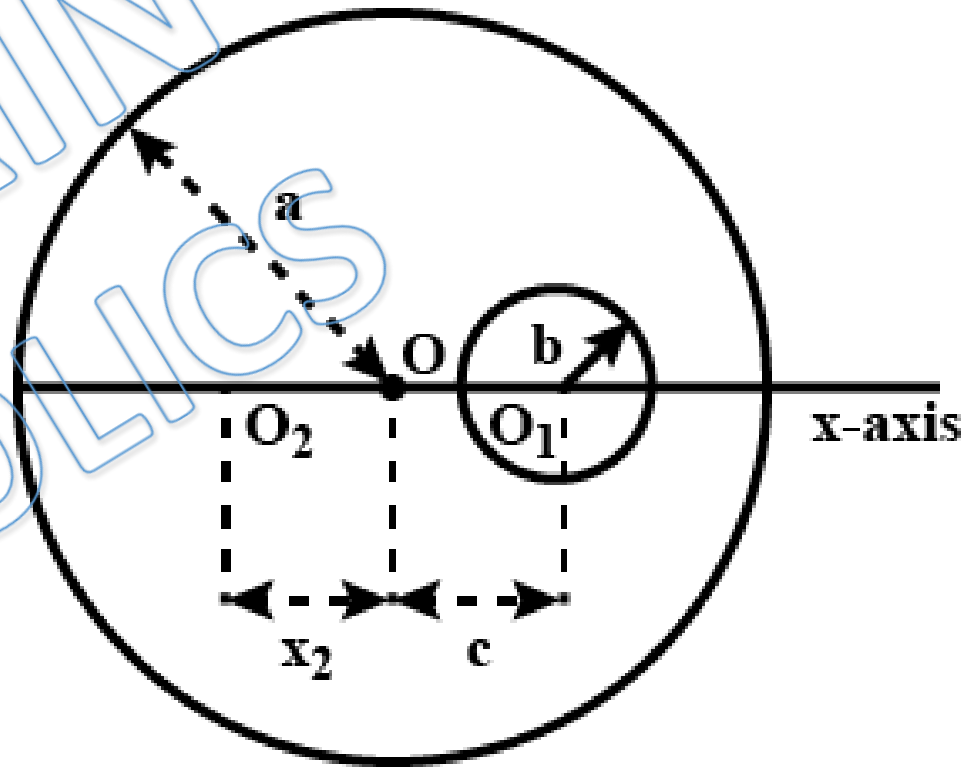
When; m_1 mass is removed from M

$$x_{cm} = \frac{M x - m_1 x_1}{M - m_1} = \frac{k\pi a^2(0) - k\pi b^2(c)}{k\pi a^2 - k\pi b^2}$$

$$x_{cm} = \frac{-b^2 c}{a^2 - b^2}$$

$$x_2 = |x_{cm}|$$

$$\Rightarrow \boxed{x_2 = \frac{cb^2}{a^2 - b^2}} \text{ Ans.}$$



Ans. b

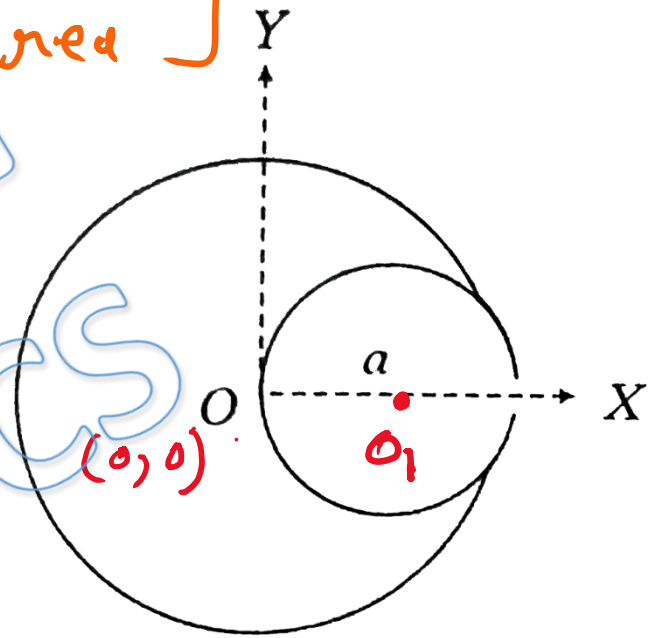
Solution: 6

COM of removed disc $O_1 : \left(\frac{a}{2}, 0\right)$

Mass of removed disc $m_1 = k\pi\left(\frac{a}{2}\right)^2$ [where, k = mass of disc in unit area]

Total mass of disc $M = k\pi a^2$

COM of total disc $O : (0, 0)$



$\therefore$ COM of remaining portion :

$$x_{cm} = \frac{M(0) - m_1\left(\frac{a}{2}\right)}{M - m_1} = \frac{0 - \left(k\pi\frac{a^2}{4}\right)\left(\frac{a}{2}\right)}{\left(k\pi a^2 - k\pi\frac{a^2}{4}\right)}$$

$$x_{cm} = \frac{-a/8}{1 - 1/4} = \frac{-a/8}{3/4} = -\frac{a}{6} \Rightarrow \boxed{x_{cm} = -\frac{a}{6}}$$

$$y_{cm} = \frac{M(0) - m_1(0)}{M - m_1} = 0 \Rightarrow \boxed{y_{cm} = 0}$$

$\therefore$ COM of remaining disc $\equiv \boxed{(x_{cm}, y_{cm}) \equiv \left(-\frac{a}{6}, 0\right)}$ Ans

Ans. d

Solution: 7

Length of element; $dL = R\,d\theta$

$$dm = \frac{M}{L}dL = \frac{M}{R\left[\frac{\pi}{2}\right]}R\,d\theta$$

$$\boxed{dm = \frac{2M}{\pi}d\theta}$$

$x = r\cos\theta$

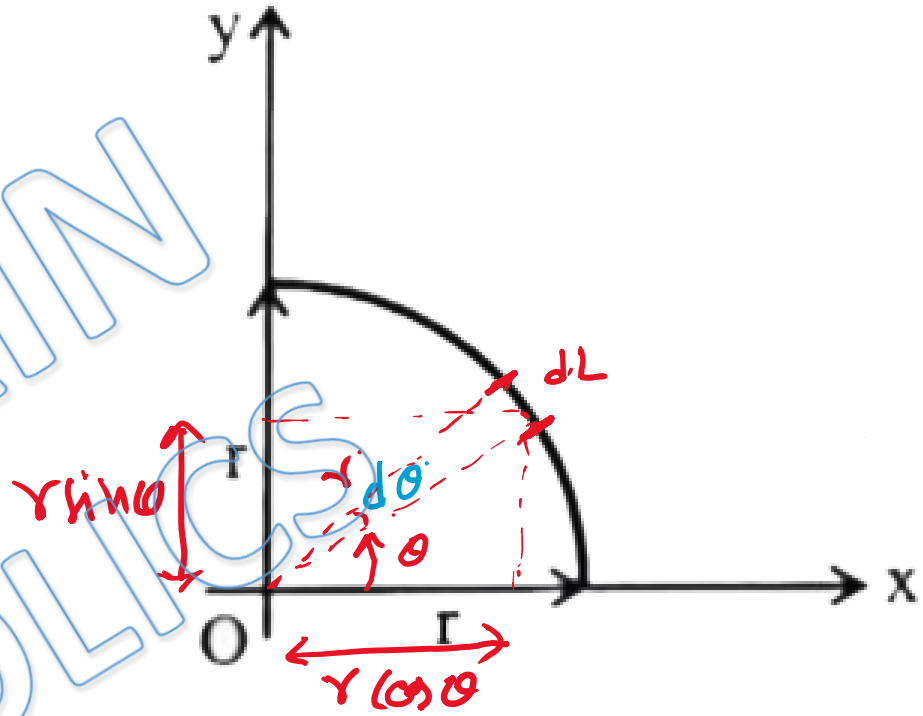
$y = r\sin\theta$

$$x_{cm} = \frac{\int dm\,x}{\int dm} = \frac{\int_0^{\pi/2}\left(\frac{2M}{\pi}d\theta\right)(r\cos\theta)}{\int_0^{\pi/2}\left[\frac{2M}{\pi}d\theta\right]}$$

$$x_{cm} = \frac{\int_0^{\pi/2}(r\cos\theta)\,d\theta}{\int_0^{\pi/2}d\theta} = \frac{r\left[\sin\theta\right]_0^{\pi/2}}{\left[\theta\right]_0^{\pi/2}} = \frac{r\left[\sin\frac{\pi}{2} - \sin 0\right]}{\frac{\pi}{2} - 0} = \frac{r[1-0]}{\pi/2}$$

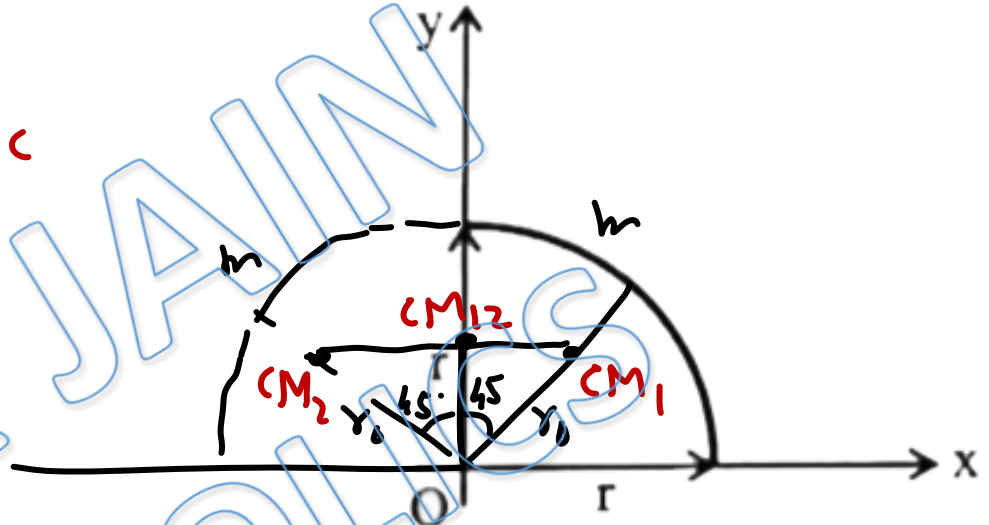
$$\Rightarrow \boxed{x_{cm} = \frac{2r}{\pi}}$$

Similarly; $\boxed{y_{cm} = \frac{2r}{\pi}}$ Ans.



Ans. c

add identical quarter Circular Arc to make a semicircular arc.



CM_1 is CM of old arc.

CM_2 " " " new arc

CM_{12} " " " Semicircular Arc.

Distance of CM_{12} from $O = \frac{2r}{\pi} = \gamma_0 \cos 45^\circ$

$\Rightarrow \gamma_0 = \frac{2\sqrt{2}\, r}{\pi}$

Position of $CM_1 = \left(\frac{\gamma_0}{\sqrt{2}}, \frac{\gamma_0}{\sqrt{2}}\right) = \left(\frac{2r}{\pi}, \frac{2r}{\pi}\right)$

Ans. c

Solution: 8

ρ_s = density of sphere = ρ ; $m_s = \rho\left(\frac{4}{3}\pi(r)^3\right) = m$

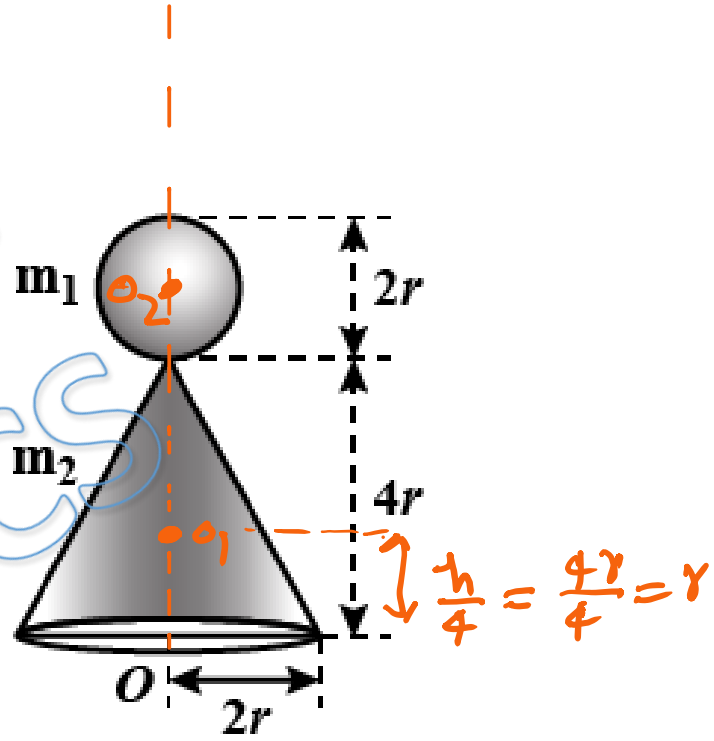
ρ_c = density of cone = $\frac{\rho}{12}$; $m_c = \frac{\rho}{12}\left(\frac{1}{3}\pi(2r)^2 \times (4r)\right) = \frac{m}{3}$

$$y_{cm} = \frac{m_s y_s + m_c y_c}{m_s + m_c}$$

$$y_{cm} = \frac{m \times 5r + \frac{m}{3} \times \left(\frac{4r}{4}\right)}{m + m/3} = \frac{16mr/3}{4m/3}$$

$y_{cm} = 4r$

$\boxed{y_{cm} = 4r}$ Ans.

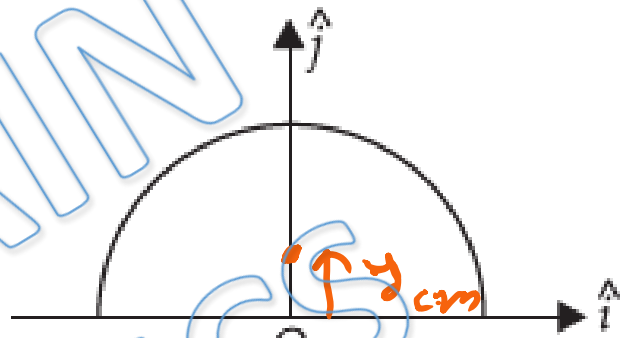


Ans. a

Solution: 9

COM of semicircular ring

$$y_{cm} = \frac{2r}{\pi}$$



so, $\boxed{(x_{cm}, y_{cm}) : \left(0, \frac{2r}{\pi}\right)}$ Ans.

Ans. c

Solution: 10

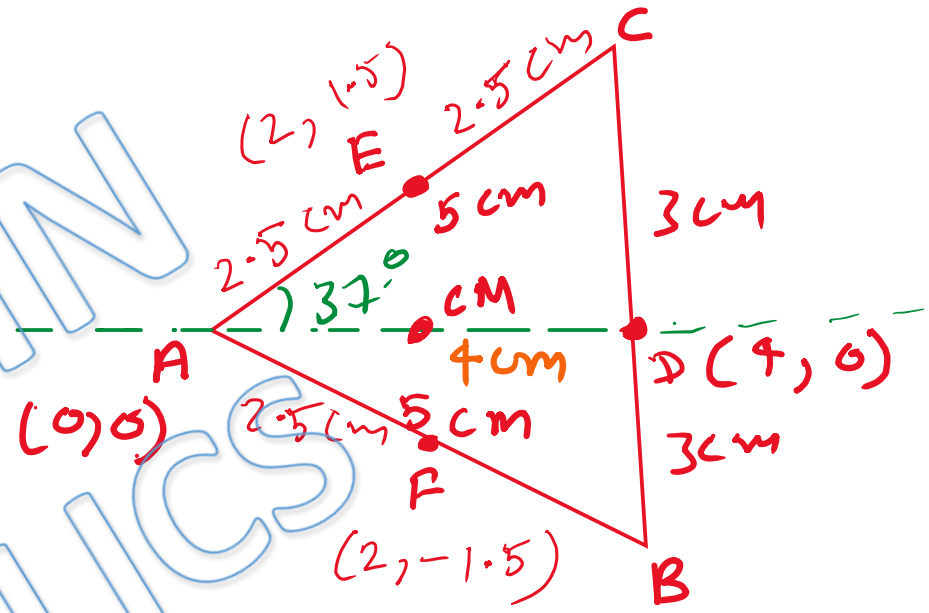
In ΔADC

$\angle A = 37^\circ$ & $\angle ADC = 90^\circ$

So; $AD = 4$ cm

$\rho_{AB} = \rho_{AC} = \rho$

and $\rho_{BC} = 2\rho$



com of Rod BC = D (4, 0)

com of Rod AC = E (2.5 Cos37°, 2.5 Sin37°)

= E (2, 1.5)

similarly; com of Rod AB = F (2, -1.5)

mass of Rod AB or AC = $m_{AB} = m_{AC} = \rho(4\times5)$

mass of Rod BC = $m_{AC} = 2\rho(4\times6)$

$$x_{cm} = \frac{(m_{AB}+m_{AC})x_1 + m_{CB}\times x_2}{m_{AB}+m_{AC}+m_{BC}} = \frac{(2\times\rho4\times5)(2) + (2\rho(4\times6)(4)}{2\rho4\times5 + 2\rho4\times6} = \frac{10+24}{11}$$

$$x_{cm} = \frac{34}{11}\ \text{cm}$$

$$y_{cm} = \frac{(\rho4\times5)(1.5) + (\rho4\times5)(-1.5) + (2\rho4\times6)\times(0)}{2\rho4\times5 + 2\rho4\times6} = 0 \Rightarrow y_{cm} = 0$$

distance from A = x_{cm}

$$d = \frac{34}{11}\ \text{cm}$$

Ans. d

Ans

Solution: 11

C = COM of complete cone

C_1 = COM of upper cone

C_2 = COM of frustum

Total mass of cone $= \rho\left(\frac{1}{3}\pi R^2\right)\times(2H) = M$

Mass of upper cone $= m_2 = \rho\left(\frac{1}{3}\pi\left(\frac{R}{2}\right)^2\right)\times H$

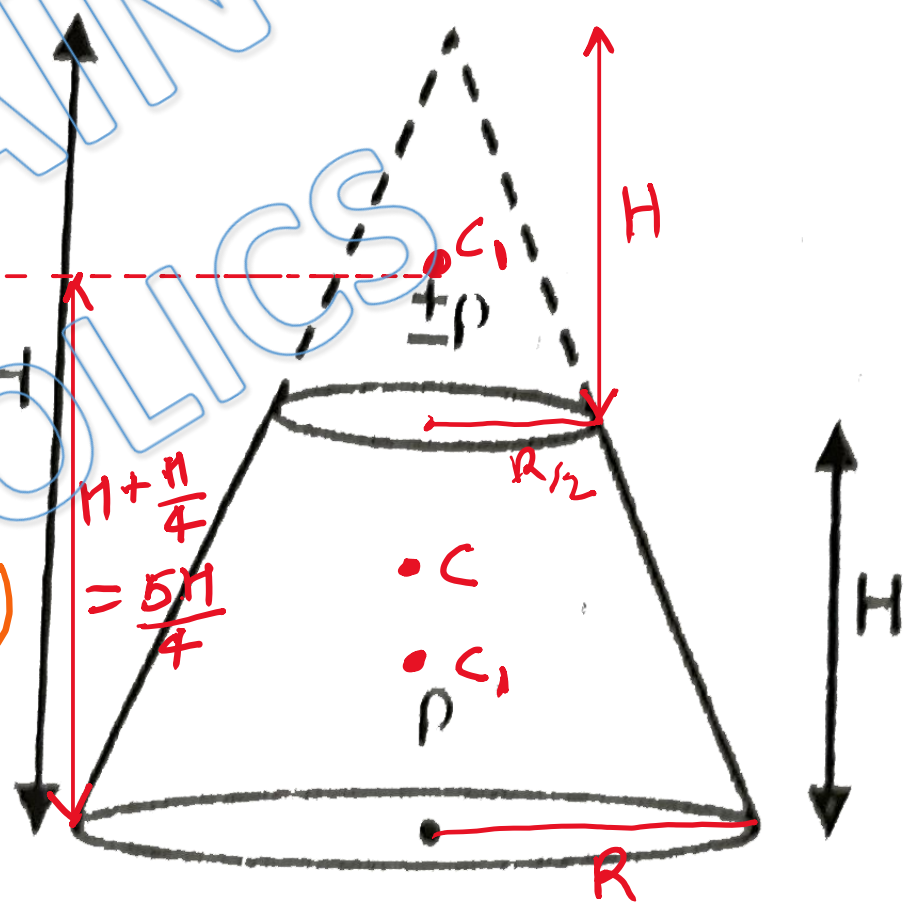
So; y_{cm} for frustum :

$$y_{cm} = \frac{My - m_2 y_2}{M - m}$$

$$y_{cm} = \frac{\rho\left(\frac{1}{3}\pi R^2\right)\times 2H \times\left(\frac{2H}{4}\right) - \rho\left(\frac{1}{3}\pi\left(\frac{R}{2}\right)^2\right)\times H\times\left(\frac{5H}{4}\right)}{\rho\left(\frac{1}{3}\pi R^2\right)\times 2H - \rho\left(\frac{1}{3}\pi\left(\frac{R}{2}\right)^2\right)\times H}$$

$$y_{cm} = \frac{H - \frac{5H}{16}}{2 - \frac{1}{4}} = \frac{\frac{11H}{16}}{7/4} = \frac{11H}{28}$$

$\boxed{y_{cm} = \frac{11H}{28}} \Rightarrow \frac{11H}{28} = \frac{11H}{4n} \Rightarrow n = \frac{28}{4} = 7 \Rightarrow \boxed{n = 7}$ Ans



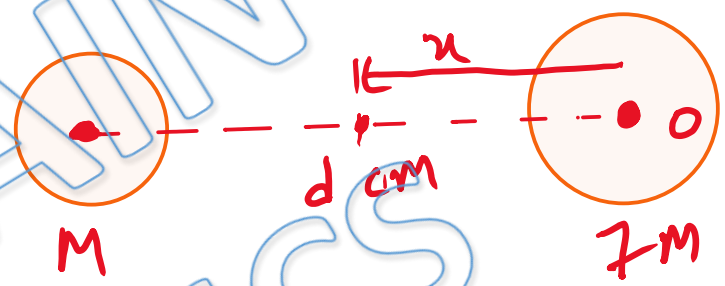
Ans. c

Solution: 12

$$x = \frac{7M(0) + M(d)}{7M + M}$$

$$x = \frac{Md}{8M}$$

$$\boxed{x = \frac{d}{8}}$$ Ans



Ans. a

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